composite functions

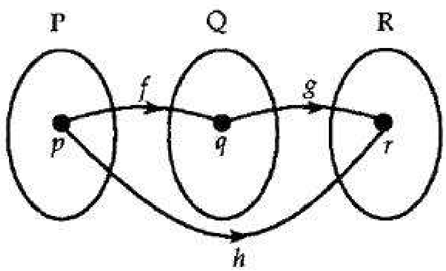
[SQA] 1. $f(x) = 3 - x$ and $g(x) = \frac{3}{x}, x \neq 0$.

(*a*) Find $p(x)$ where $p(x) = f(g(x))$. **2**

(*b*) If $q(x) = \frac{3}{3-x}, x \neq 3$, find $p(q(x))$ in its simplest form. **3**

[SQA] 2. The diagram illustrates three functions f, g and h. The functions are defined by $f(x) = 2x + 5$ and $g(x) = x^2 - 3$.

The function h is such that whenever $f(p) = q$ and $g(q) = r$ then $h(p) = r$.



(*a*) If $q = 7$, find the values of p and r. **2**

(*b*) Find a formula for $h(x)$, in terms of x. **2**

[SQA] 3. On a suitable set of real numbers, functions f and g are defined by $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{1}{x} - 2$.

Find $f(g(x))$ in its simplest form. **3**

[SQA] 4. $f(x) = 2x - 1$, $g(x) = 3 - 2x$ and $h(x) = \frac{1}{4}(5 - x)$.

(*a*) Find a formula for $k(x)$ where $k(x) = f(g(x))$. **2**

(*b*) Find a formula for $h(k(x))$. **2**

(*c*) What is the connection between the functions h and k? **1**

[SQA] 5. A function f is defined on the set of real numbers by $f(x) = \frac{x}{1-x}, x \neq 1$.

Find, in its simplest form, an expression for $f(f(x))$. **3**

[SQA] 6. The functions f and g, defined on suitable domains, are given by $f(x) = \frac{1}{x^2 - 4}$ and $g(x) = 2x + 1$.

(*a*) Find an expression for $h(x)$ where $h(x) = g(f(x))$. Give your answer as a single fraction. **3**

(*b*) State a suitable domain for h. **1**

[SQA] 7. Functions f and g, defined on suitable domains, are given by $f(x) = 2x$ and $g(x) = \sin x + \cos x$.

Find $f(g(x))$ and $g(f(x))$. **4**

[SQA] 8. The functions f and g are defined on a suitable domain by $f(x) = x^2 - 1$ and $g(x) = x^2 + 2$.

(*a*) Find an expression for $f(g(x))$. **2**

(*b*) Factorise $f(g(x))$. **2**

[SQA] 9. Functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = \dfrac{x^2 + 25}{x^2 - 25}$ where $x \in \mathbb{R}$, $x \neq \pm 5$.

The function h is given by the formula $h(x) = g(f(x))$.

For which real values of x is the function h **undefined**? **4**

[SQA] 10. Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

(*a*) (i) Find $p(x)$ where $p(x) = f(g(x))$.

(ii) Find $q(x)$ where $q(x) = g(f(x))$. **3**

(*b*) Solve $p'(x) = q'(x)$. **3**

11. Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$.

(*a*) Find $g(f(x))$. **2**

(*b*) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$. **1**

(*c*) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.

(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully. **5**

(*d*) Hence solve $g(f(x)) + xh(x) = 0$. **1**

[SQA] 12.

(*a*) The function f is defined by $f(x) = x^3 - 2x^2 - 5x + 6$.

The function g is defined by $g(x) = x - 1$.

Show that $f\big(g(x)\big) = x^3 - 5x^2 + 2x + 8$. **4**

(*b*) Factorise fully $f\big(g(x)\big)$. **3**

(*c*) The function k is such that $k(x) = \dfrac{1}{f\big(g(x)\big)}$.

For what values of x is the function k not defined? **3**

[SQA] 13. (*a*) $f(x) = 2x + 1$, $g(x) = x^2 + k$, where k is a constant.

(i) Find $g(f(x))$. **(2)**

(ii) Find $f(g(x))$. **(2)**

(*b*) (i) Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to $2x^2 + 4x - k = 0$. **(2)**

(ii) Determine the nature of the roots of this equation when $k = 6$. **(2)**

(iii) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots. **(3)**

[SQA] 14. Functions f and g are defined on the set of real numbers by $f(x) = x - 1$ and $g(x) = x^2$.

(*a*) Find formulae for

(i) $f\big(g(x)\big)$

(ii) $g\big(f(x)\big)$. **4**

(*b*) The function h is defined by $h(x) = f\big(g(x)\big) + g\big(f(x)\big)$.

Show that $h(x) = 2x^2 - 2x$ and sketch the graph of h. **3**

(*c*) Find the area enclosed between this graph and the x-axis. **4**

[SQA] 15. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(a) Find expressions for:

(i) $f(h(x))$;

(ii) $g(h(x))$. **2**

(b) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$.

(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \leq x \leq 2\pi$. **5**

[SQA] 16. Functions f and g are defined on suitable domains by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$.

(a) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **2**

(b) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

[END OF QUESTIONS]